Fall 2014

Optics

Part 1: Round trip phase delay: $\delta_1 = \frac{4\pi\nu_1 l}{c} + 2\phi_{r1} = m\pi$

$$\delta_2 = \frac{4\pi\nu_2 l}{c} + 2\phi_{r2} = m\pi$$

Equating δ_1 & δ_2: $\frac{4\pi(\nu_1-\nu_2)l}{c} + 2(\phi_{r1}-\phi_{r2}) = 0$

$$(\phi_{r1}-\phi_{r2}) = \frac{2\pi l}{c}(\nu_1-\nu_2) =$$

$$\frac{2\pi(1.5)}{3\times10^8}(10^7) = 2\pi(0.05)$$

$$\therefore\ a\lambda = 0.05\lambda = \lambda/20$$

Alternatively: A $\frac{\lambda}{2}$ plate would produce coincident mode frequencies $l = \frac{m\lambda_1}{2} = \frac{(m+1)\lambda_2}{2}$

$\therefore$ $\frac{\lambda}{2}$ retardation produces $\Delta\nu = \frac{c}{2l} = 100$ MHz.

For $\Delta\nu = 10$ MHz, we must have

$$\boxed{a\lambda = \left(\frac{10}{100}\right)\left(\frac{\lambda}{2}\right) = \frac{\lambda}{20}}$$

Part 2:

At low power, $I(\nu_0) = I_0 \exp[-\alpha(\nu_0) l]$

$$\Rightarrow \alpha(\nu_0) l = 0.2 \quad \text{(Power absorption)}$$

But $E = E_0 \exp[-a(\nu_0) + j\beta(\nu_0)] l$ [amplitude]

$$\text{where } \underbrace{a(\nu_0)}_{\text{Amp.}} = \frac{\alpha(\nu_0)}{2} \leftarrow \text{Intensity}$$

$$\therefore a(\nu_0) l = 0.1$$

We have $\frac{k\chi''(\nu) l}{2n^2} = -a(\nu_0) l = 0.1$

From Kramers-Kronig relations, $\chi' = \frac{\chi'' 2(\nu_0 - \nu)}{\Delta\nu}$

$$\Rightarrow \chi'(\nu_c) = \chi''(\nu_c) \frac{2(\nu_0 - \nu_c)}{\underbrace{2(\nu_0 - \nu_c)}_{\text{Def of } \Delta\nu}} = \chi''(\nu_c)$$

Also, $\chi''(\nu_c) = \frac{\chi''(\nu_0)}{2}$ (by def. of ν_c)

$$\therefore \beta(\nu_c) l = \frac{k\chi'(\nu_c) l}{2n^2} = \frac{k\chi''(\nu_c) l}{2n^2} = \frac{k\chi''(\nu_0) l}{4n^2} = \frac{\alpha(\nu_0) l}{2} = 0.05$$

This corresponds to a single pass delay. For the round-trip delay, $\delta = 2(0.05) = 0.1$. This is the phase shift at low power. When the transition saturates, this phase shift goes to zero, and detunes the Fabry-Perot

$$T = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(\delta/2)} = \frac{(1-0.9)^2}{(1-0.9)^2 + 4(0.9)(0.05)^2} = 0.52$$